

REFLECTION OF ULTRASONIC WAVES BY AN IMPERFECT DIFFUSION BOND

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INTRODUCTION

The analytical treatment of the reflection of ultrasonic wave motion by a planar distribution of cracks is of interest for the non-destructive evaluation of imperfect diffusion bonds. Preliminary results for an experimental approach have been given by Hosten et al.¹, for two bonded stainless steel cylinders. In practice, new high strength steel tubing has complicated the pinch welding process and placed emphasis on the integrity of the resulting weld, see Rehbein et al.², Thomas et al.³. Two ultrasonic nondestructive evaluation techniques to find defects in the pinch weld and to determine weld strength have been discussed by Thomas et al.⁴.

In this paper we are interested in the reflection and transmission of a longitudinal wave by a distribution of cracks located in a plane in the interior of a solid body. Reflection and transmission coefficients for a planar array of periodically spaced cracks have been calculated by Angel and Achenbach^{5,6}. Here a novel method is developed which is approximate, but has broader applicability. The method is based on the calculation of crack-opening volumes for any number of cracks under the action of a normally incident longitudinal wave. The effect of a neighboring crack is represented by a pair of dipoles at the geometrical center of that crack, whose strengths are proportional to the crack-opening volume. This approach allows a relatively simple calculation, which is verified by comparison with exact results, and which shows that often only the next nearest cracks have an appreciable influence. For a plane with many cracks a simple expression has been derived for the reflection and transmission coefficients in terms of the number of cracks per unit length and the crack-opening volumes.

REFLECTION AND TRANSMISSION COEFFICIENTS

The bond plane of two bodies of the same homogeneous, isotropic, linearly elastic material is assumed to contain a large number of cracks. The plane is defined by $x_2 = 0$, and the cracks extend to infinity in the x_3 -direction. As shown in Fig. 1, we consider a cell defined by $|x_2| \leq h$, $|x_1| \leq \ell/2$, which contains N cracks. The length of the i -th crack is $2a_i$, while the distance between the centers of the i -th and $(i+1)$ -th cracks is D_i . We further consider normal incidence on the cracks of a plane time-harmonic longitudinal wave, $u_I \exp(-i\omega t)$,

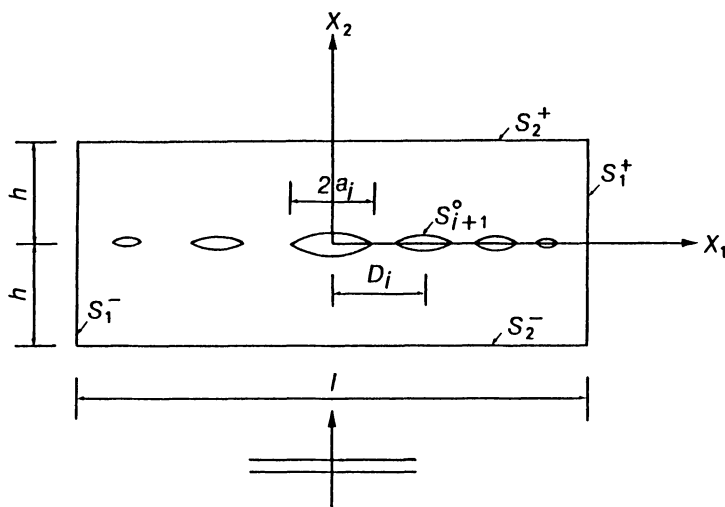


Fig. 1 Array of cracks in the plane $x_2 = 0$.

where \underline{u}^I is given by

$$\underline{u}^I = u_0 \underline{p} \exp(ik_L \underline{p} \cdot \underline{x}), \quad \underline{p} = (0, 1), \quad (1, 2)$$

in which $k_L = \omega/c_L$, $c_L^2 = (\lambda + 2\mu)/\rho$. The amplitude u_0 and the angular frequency ω characterize the incident plane wave, while the Lamé constants, λ, μ , and the density, ρ , characterize the elastic solid. The multiplicative factor $\exp(-i\omega t)$, common to all field variables, is omitted in the sequel.

It is assumed that far from the plane of the cracks the dominant parts of the displacement fields are plane longitudinal waves, which may be written as

$$x_2 < 0: \quad \underline{u}^-(\underline{x}) = u_0 \underline{p} \exp(ik_L \underline{p} \cdot \underline{x}) + u_0 \underline{p}^- R \exp(ik_L \underline{p}^- \cdot \underline{x}) \quad (3)$$

$$x_2 > 0: \quad \underline{u}^+(\underline{x}) = u_0 \underline{p} T \exp(ik_L \underline{p} \cdot \underline{x}), \quad \underline{p}^- = (0, -1), \quad (4, 5)$$

where R and T denote the reflection and transmission coefficients respectively.

To determine expressions for the coefficients R and T , we now apply the reciprocal identity to the cell defined by $|x_1| \leq l/2$, $|x_2| \leq h$ ($h \rightarrow \infty$), shown in Fig. 1. We have

$$\int_S (\underline{u}_i^B \underline{t}_i^A - \underline{u}_i^A \underline{t}_i^B) dS = 0 \quad (6)$$

where \underline{u}^A and \underline{u}^B are two elastodynamic displacement states with zero body forces, and \underline{t}^A and \underline{t}^B are the corresponding tractions. Also

$$S = S_1^+ + S_2^+ + \sum_{i=1}^N S_i^0 \quad (7)$$

Here, S_1^+, S_2^+ are the exterior bounding surfaces of the cell, as shown in Fig. 1, and S_i^0 is the surface bounding the i -th crack. For $\{\underline{u}_i^A, \underline{t}_i^A\}$ we choose the actual state corresponding to Eqs. (3) and (4), while for

$\{u_i^B, t_i^B\}$ the following two wave states are chosen:

$$u_R^B = -u_0 p^- \exp(-ik_L p^- \cdot x), \quad u_T^B = -u_0 p \exp(-ik_L p \cdot x), \quad (8,9)$$

which are present both in $x_2 < 0$ and $x_2 > 0$. These auxiliary wave states are opposite in phase and propagate in directions opposite to the reflected and transmitted waves of Eqs.(3) and (4), respectively. Substitution of Eqs.(3) and (4) into Eq.(6) for state A, and Eq.(8) for state B, gives

$$R = \left[\frac{1}{N} \sum_{i=1}^N V^i \right] \left[\frac{N}{\ell} \right] \left[\frac{1}{2u_0} \right] \quad (10)$$

where

$$V^i = \int_{-a_i}^{a_i} \Delta u_2^i(\zeta_1) d\zeta_1, \quad (11)$$

and

$$\Delta u_2^i(\zeta_1) = u_2^{Ai}(\zeta_1, 0^-) - u_2^{Ai}(\zeta_1, 0^+). \quad (12)$$

Equation (12) represents the crack-opening displacement, while Eq.(11) represents the crack-opening volume per unit length in the x_3 -direction. In deriving (10), we used the boundary condition for the crack, i.e., $t_2^A = 0$ on S_1^0 . In a similar manner, using Eq.(9) for state B, we obtain the transmission coefficient as $T = 1 + R$, where R is given by (10).

Equation (10) expresses the reflection coefficient as a product of the average crack-opening volume times the average number of cracks per unit length. Since N is large, we can write

$$\ell \approx \sum_{i=1}^N D^i, \quad (13)$$

which implies that the average number of cracks per unit length is equal to the inverse of the average separation distance of the cracks. For an infinite array of periodically spaced cracks of equal length, $D^i = D$, $V^i = V$, and Eqs.(10) and (13) give

$$R = \frac{V}{2Du_0}. \quad (14)$$

CRACK-OPENING VOLUMES FOR TWO CRACKS

We consider normal incidence of a plane longitudinal wave on two coplanar cracks with lengths $2a_i$ ($i = 1, 2$), and with distance D between their geometrical centers. The cracks lie in the plane $x_2 = 0$, and extend to infinity in the x_3 -direction. The integral equation for the crack-opening displacement has been obtained and solved numerically by Zhang and Achenbach⁷ by the Boundary Integral Equation Method. Here, a different method of solution is proposed that proves extremely useful to solving the problem of an array of cracks (deterministic or probabilistic) that would otherwise be extremely cumbersome to solve. The method can also be applied to the three-dimensional problem.

The starting point is to decouple the effects of the incident field and of the other crack on the opening of crack-i, by writing the crack-opening displacement (COD) as

$$\Delta u_2^i(x_1) = \Delta u_2^{i0}(x_1) + \Delta u_2^{ij}(x_1), \quad i, j = 1, 2, i \neq j \quad (15)$$

In Eq.(15), $\Delta u_2^{10}(x_1)$ is the COD of crack-1 induced by the incident field in the absence of the other crack, and $\Delta u_2^{1j}(x_1)$ is the COD induced by the presence of crack-j. Substitution of (15) into the integral representation for the scattered displacement, subsequent substitution of the result in Hooke's law, and application of the boundary conditions on the crack faces then yields for crack 1 ($|x_1| \leq a$, $x_2 = 0$):

$$\int_{-a_1}^{a_1} L(x_1, \xi) \Delta u_2^{10}(\xi_1) d\xi_1 = -\sigma_{22}^I \quad (16)$$

and

$$\int_{-a_1}^{a_1} L(x_1, \xi) \Delta u_2^{12}(\xi_1) d\xi_1 = - \int_{D-a_2}^{D+a_2} L(x_1, \xi) \Delta u_2^2(\xi_1) d\xi_1 \quad (17)$$

where

$$L(x_1, \xi) = \lim_{x_2 \rightarrow 0} C_{22\alpha\beta} \frac{\partial}{\partial x_\beta} \sigma_{22\alpha}^G(x-\xi) \quad (18)$$

Equation (16) follows from the definition of Δu_2^{10} . The term $\sigma_{22\alpha}^G(x-\xi)$ is the stress σ_{22} produced at x due to a unit harmonic line load applied at ξ and pointed in the x_α direction.

In the second step of our method the effect of the continuous distribution of dipoles of crack-2 (right hand side of Eq.(17)), is approximated by the effect of a single pair of dipoles (1-1,2-2) of respective strengths λV^2 and $(\lambda+2\mu)V^2$, located at the geometrical center of crack-2. The crack opening volume V^2 , which is defined by Eq.(11), is a complex quantity which is not known a-priori. Since V^2 is a multiplicative factor we shall first obtain the crack-opening displacement due to the pair of dipoles of strengths λ and $\lambda+2\mu$. This is achieved by solving numerically Eq.(17) with V^2 being real and equal to unity. The solution follows the method of Zhang and Achenbach⁷ who obtained the COD of a single crack due to a general load.

Next, we calculate the integral

$$\alpha_2^1 = \int_{-a_1}^{a_1} \Delta u_2^{12} d\xi_1 \quad (19)$$

The corresponding quantity for crack-2 can be computed in the same manner. In general terms we define α_j^i , as the approximate complex crack-opening volume of crack-i due to a unit real crack-opening volume of crack-j. The crack-opening volume of crack-i is therefore obtained approximately by the superposition

$$V^i \approx V_o^i + \alpha_j^i V^j, \quad i, j = 1, 2, i \neq j \quad (20)$$

where V_o^i is the opening volume of crack-i induced by the incident field in the absence of the other crack and obtained from the solution of (16). Clearly, V^j is given by (20) with i and j interchanged. Simultaneous solution of the two algebraic equations defined by (20) gives

$$V^i = \frac{V_o^i + \alpha_j^i V_o^j}{1 - \alpha_j^i \alpha_i^j}, \quad i, j = 1, 2, i \neq j \quad (21)$$

For the particular case that the cracks have the same length ($\alpha_2^1 = \alpha_1^2 = \alpha$, $V_o^1 = V_o^2 = V_o$), Eq.(21) reduces to

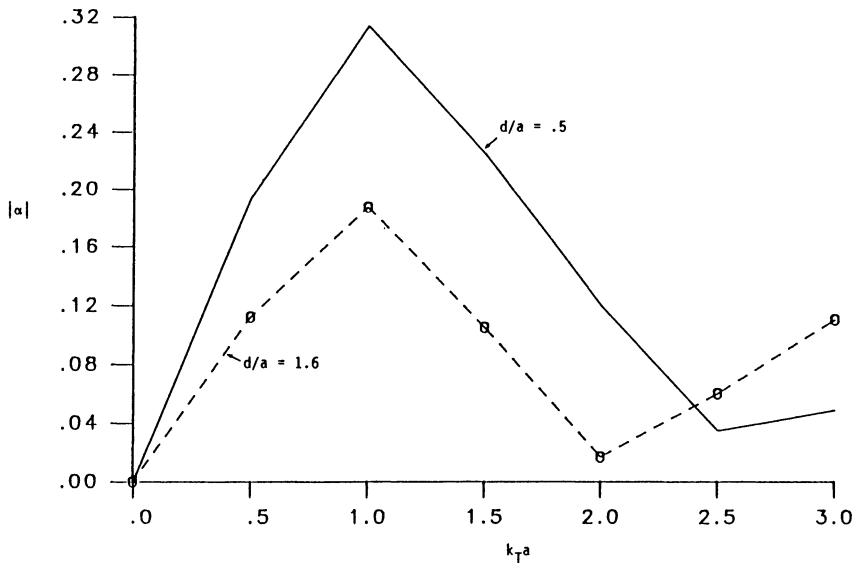


Fig. 2 Amplitude of interaction factor α versus $k_T a$ for $d/a = 1.6$ and $d/a = 0.5$ ($d/a = D/a - 2$).

$$V = \frac{V_0}{1-\alpha} \quad (22)$$

We now present a plot that exhibits the variation of α with the frequency parameter $k_T a$ and the geometrical parameter d/a . This plot not only shows the interaction of dipoles with a crack, but also sheds light on the solution for an array of cracks. In Fig. 2, the amplitude of α is shown as a function of $k_T a$, for $d/a = 1.6$ and for $d/a = 0.5$ ($d/a = D/a - 2$). It is observed that the maximum amplitude of α occurs in the neighborhood of $k_T a = 1$, virtually independent of the separation distance, D/a , between the dipole and the center of the crack.

The proposed approximate solution given by Eq.(22) has been compared with the "exact" solution obtained numerically by the method of Zhang and Achenbach⁷. It was concluded that our approximation is valid for $d/a \geq 0.5$ and for all frequencies. It was also found that for $d/a \geq 4$, a crack opening volume can be obtained with satisfactory accuracy by ignoring the existence of a neighboring crack.

CRACK-OPENING VOLUMES FOR AN ARRAY OF CRACKS

Our method is now extended to normal incidence on N cracks, where $N > 2$. The crack-opening volume of crack- i is written as

$$V^i = V_o^i + V_c^i, \quad i = 1, 2, \dots, N \quad (23)$$

where V_c^i is the opening volume of crack- i induced by the presence of all the other cracks. We now propose the following approximation

$$V^i = V_o^i + \sum_{\substack{j=1 \\ j \neq i}}^N \alpha_j^i V^j, \quad i = 1, 2, \dots, N \quad (24)$$

where V^i and α_j^i are determined, as explained before. Equation (24) can be further simplified, since α_j^i is negligible for separation distances, d_i/a_j , of the order of four. This gives

$$V^i = V_o^i + \alpha_{i-1}^i V^{i-1} + \alpha_{i+1}^i V^{i+1}, \quad i = 1, 2, \dots, N \quad (25)$$

Equation (25) expresses the opening volume of crack- i as the sum of three terms. Its opening volume, V_o^i , as if there were no other cracks, plus an opening volume caused by two pairs of dipoles, each pair located at the center of the nearest crack on each side. For the two cracks positioned at the ends of an array, obviously only one pair of dipoles is effective. The generalized moments of the pairs of dipoles are λV^{i-1} , $(\lambda + 2\mu)V^{i-1}$ and λV^{i+1} , $(\lambda + 2\mu)V^{i+1}$.

The N linear algebraic equations represented by (25) can be solved simultaneously in a straight-forward manner. The solution gives the crack-opening volume, V^i ($i = 1, 2, \dots, N$) of each crack. When the number of cracks is large it is cumbersome to solve (25). Thus we shall obtain approximately the opening volume of crack- i without solving simultaneously all of the N equations of (25). If we set $V_{i-1}^{i-1} = V_o^{i-1}$ and $V_{i+1}^{i+1} = V_o^{i+1}$ in the i th equation of (25) we have

$$V^i \approx V_o^i + \alpha_{i-1}^i V_o^{i-1} + \alpha_{i+1}^i V_o^{i+1}, \quad (26)$$

In other words as far as crack- i is concerned only its nearest two neighboring cracks have opened. Each one opens as much as when no other crack is present. If instead we take into account in V_{i-1}^{i-1} and V_{i+1}^{i+1} the effects of their neighboring cracks as we did for V^i in (26) we get

$$\begin{aligned} V^i \approx V_o^i (1 + \alpha_{i+1}^i \alpha_i^{i+1} + \alpha_{i-1}^i \alpha_i^{i-1}) + \alpha_{i-1}^i V_o^{i-1} + \alpha_{i+1}^i V_o^{i+1} \\ + \alpha_{i+1}^i \alpha_{i+2}^{i+1} V_o^{i+2} + \alpha_{i-1}^i \alpha_{i-2}^{i-1} V_o^{i-2}. \end{aligned} \quad (27)$$

Improvement on the accuracy of V^i given by (27) can be obtained by repetition of this process. However, we concluded that $2|\alpha|$ is small. Thus, after a very small number of successive substitutions a very satisfactory solution can be obtained. A crude first approximation of V^i is given by V_o^i . Better approximations of V^i are given successively by (26), (27) etc.

As a special case, we consider an infinite number of cracks periodically spaced and of equal length. Since V_o and α are the same for all the cracks, V is also the same, and any of (25) gives

$$V = \frac{V_o}{1-2\alpha}. \quad (28)$$

The approximation (27), for this case gives

$$V \approx V_o [1 + 2\alpha + 4\alpha^2], \quad (29)$$

which happen to be the first three terms of a binomial expansion of

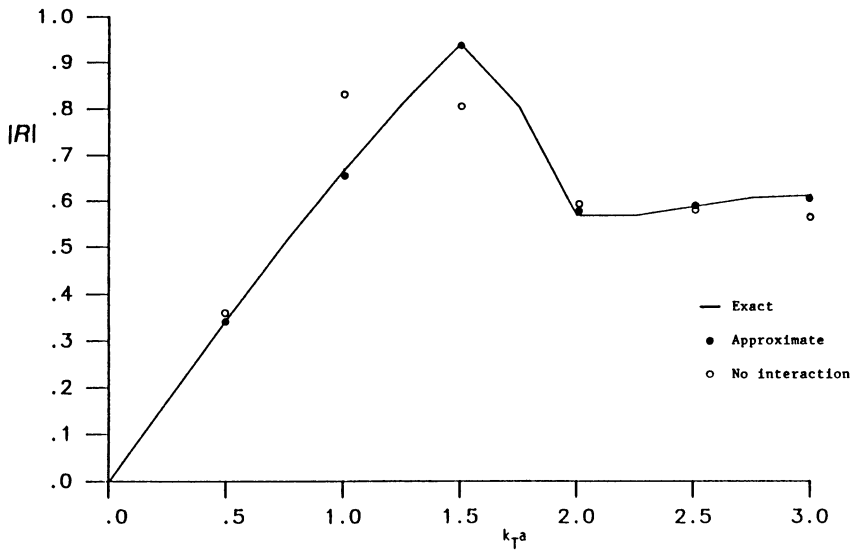


Fig. 3 Absolute value of reflection coefficient
versus $k_T a$ for $d/a = 1.6$.

$(1 - 2\alpha)^{-1}$ in (28). Substitution of (28) into (14) gives

$$R = \frac{V_o}{2Du_o(1-2\alpha)} \quad (30)$$

Equation (30) expresses in closed form the reflection coefficient for the periodic array of cracks in terms of: (a) the crack-opening volume V_o of a single crack produced by a normally incident plane displacement wave of amplitude u_o , and (b) the dimensionless crack-opening volume α of a single crack produced by a pair of dipoles of strengths λ and $\lambda + 2\mu$ located at a distance D - the periodicity of the array - from the geometrical center of the crack.

The rigorous reflected and transmitted fields for the periodic problem were calculated numerically by Angel and Achenbach⁵ with a method different from the one we use here. The two solutions are compared in Fig. 3. In the results of Fig. 3, the Poisson's ratio of the material is $1/3$, and the separation distance of the cracks is $d/a = 1.6$. The agreement is excellent. The result for $\alpha = 0$ (disregarding the crack-interaction) is also shown. It is observed that strong interaction occurs at $k_T a = 1$ and $k_T a = 1.5$. This agrees with our conclusion derived from Fig. 2. Numerical comparisons were also made for other separation distances. Our conclusion is that (30) is valid for $d/a \geq 1$ and for all frequencies.

In the present analysis the imperfect diffusion bond was modeled as a deterministic nonperiodic distribution of cracks, and the reflection and transmission coefficients were calculated for incident plane longitudinal waves. A discussion on the solution of the inverse problem and the average reflection coefficient for a statistical distribution of cracks can be found in a forthcoming paper by the authors.

ACKNOWLEDGMENT

This paper was written in the course of research sponsored by ONR under Contract N00014-85-K-0401.

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